During the day there was a moderate breeze from the north blowing down the valley. This became very light toward evening, and at about the same time the temperature began to fluctuate, sudden changes of 2° to 5° C. occurring frequently between 6 p. m. and the time of minimum temperature. These fluctuations are well shown in the thermograph records at Independence, Cal. (fig. 15), and in Table 15, which contains observed temperatures and humidities at Lone Pine, Cal. These observations have been referred to by Dr. Wm. R. Blair in his discussion of mountain and valley temperatures (Bull., Mt. Weather obs'y, Washington, 1914, 6: 122) and are in accord with the conclusion there reached that "there is not a stream of cool air past the slope station, but a direct convective interchange between the cool air on the slope and the free air over the valley at the same or slightly lower levels." In general, as shown in Table 15, the lower temperatures were accompanied by the higher absolute humidities.

southerly current aloft, at the same time causes the surface northerly current down the valley.

THE HORIZONTAL RAINBOW.[1]

By S. Fujiwhara.

The so-called horizontal rainbow has been reported by several scientists. Julius von Hann observed this mysterious optical phenomenon on Lake Constance, and W. R. N. Church has seen it on Loch Lomond. F. Hashimoto observed such a rainbow (or horizontal spectrum) on Lake Suwa in central Japan on the morning of November 3, 1912. On this morning the weather was very clear. He and Count A. Tanaka were then engaged in limnological observation at the shore near Kakuyū-kan. Mr. Hashimoto saw the rainbow on the water surface of the northeastern part of the lake. They at first observed the bow with their eyes at a height of 9 feet
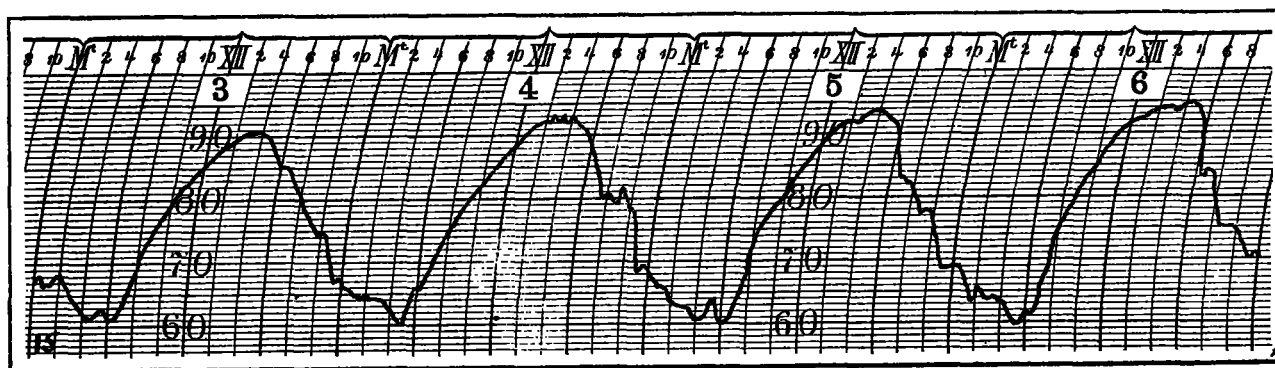

Fig. 15.—Thermograph record (°F.), at Independence, Cal., August 3–6, incl., 1913, showing fluctuations in temperature during nighttime.

TABLE 15.—*Fluctuations in surface temperature and humidity at Lone Pine, Cal., Aug. 2 and 3, 1913.*

Date.	Time.	Temperature.	Relative humidity.	Absolute humidity.
1913.	*P. m.*	*°C.*	*Per cent.*	*g/cu. m.*
Aug. 2	7:48	22.2	48	9.3
	7:51	20.6	56	9.9
	8:01	19.4	64	10.6
	8:45	20.0	56	9.6
	9:10	16.7	75	10.6
	9:21	18.7	64	10.2
	10:01	16.7	75	10.6
	11:00	18.3	62	9.6
	11:05	16.4	77	10.7
	11:48	18.9	60	9.6
Aug. 3	6:50	25.1	40	9.2
	7:40	21.1	56	10.2
	7:50	19.4	56	9.3
	8:05	20.8	45	8.1
	8:37	19.4	52	8.6
	9:09	21.1	42	7.7
	9:33	23.9	34	7.3
	9:43	21.8	47	8.9

Between 8 and 10:30 p. m. it was necessary to bring the balloon down because of southerly or southeasterly winds aloft. These winds gradually extended toward the surface and were warm and dry (Table 13). The mixing of the upper southerly and the lower northerly currents seems to account for the variations in surface temperature and humidity already referred to.

The fact that the upper southerly wind is warm and dry suggests the probability that it originates over the Mohave Desert, which is about 150 kilometers south of Lone Pine. The heating and consequent rising of air over the desert in the daytime, which gives rise to the

above the water level. On bringing their eyes down to the height of 6 feet the length of the bow diminished, but the colors became very distinct. By lowering their eyes the bow became clearer, and at last, at a height of a little lower than 4 feet, it vanished. At any height lower than this they could no more see the bow, but above this height the bow was seen. As the sun rose higher the bow shifted to the right and vanished from them at 11 a. m., while standing on the shore; soon they went up the stairs of an inn near by, and thence they could perceive the bow, though indistinctly. The position of the lake and features of the bow on this occasion were as shown in figure 1. About 8 a. m. on December 8, 1913, Mr. Hashimoto again observed a similar phenomenon on the same lake. At this time he was in a boat making limnological observations. In his letter to me he states the results of his observations. On the morning in question the surface temperature of the water of the lake was 8° C. and that of the air was about 3° C. He also observed a very thin haze or mist over the surface of the lake and the air was very calm. The optical conditions on December 8, 1913, are presented in figure 2, where the plane of the figure represents the surface of the lake water; OS' is the horizontal projection of the sun's ray passing through O, the position of the observer, Mr. Hashimoto. OV and OR are the limiting rays of the horizontal rainbow RV. The angle ROV has been estimated at about 3°, and the horizontal angle VOS at 38°. The violet side of the bow is indicated by OV and the red side by OR.

[1] *Revised and reprinted from* Jour. met. soc. Japan, Tokyo, March, 1914.

The formation of the horizontal rainbow seems to be not yet fully explained. One is naturally inclined to regard it as a phenomenon similar to the ordinary rainbow; but the only property they have in common is that of the spectrum colors. In all other respects they are quite different. The principal points of difference are:

1. The horizontal rainbow occurs in very clear weather and is not associated with rain.

2. The arrangement of the colors in a horizontal rainbow is radial, and not transverse [concentric circles] as in an ordinary rainbow.

3. The ordinary rainbow appears as a vertical circular arc having an angular radius of about 40° to 42°, while the horizontal bow or "ohikari" has the form of a horizontal narrow sector about 3° wide. Sometimes the horizontal sector is 15° wide, which is never the case in an ordinary rainbow.

THEORETICAL CONSIDERATIONS.

In the following is presented a physical explanation of this rare phenomenon.

Assume that two conditions prevailed in the actual case: (a) A thin sheet of mist formed of very small water
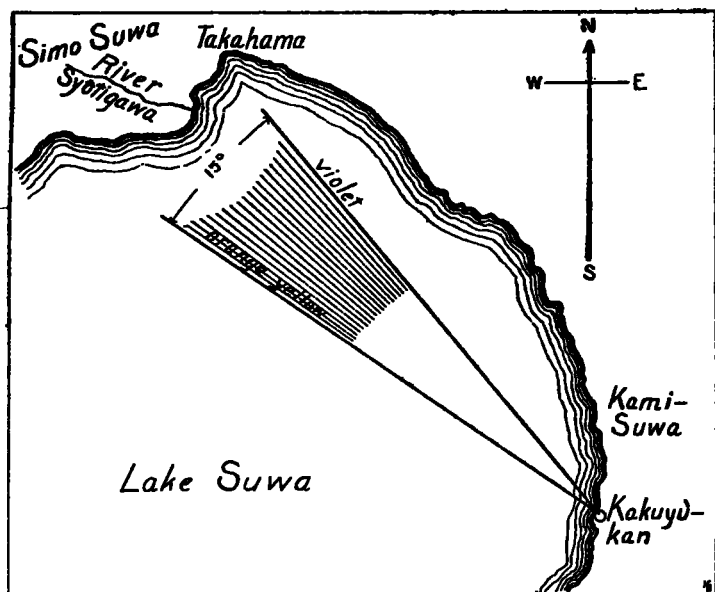

FIG. 1.—Horizontal rainbow on Lake Suwa on November 3, 1912.

drops lies over the water surface. This may often happen over any body of shallow water on a very calm and comparatively warm day. Lake Suwa is a shallow body of water; under strong insolation its water warms up readily and evaporation from the surface takes place rapidly. Mr. Hashimoto reports that on the morning of December 8, 1913, the temperature of surface water was 8°C. and that of the air above was about 3°C. He also observed very thin mist on the surface of the water. During the colder months vapor from the lake surface must condense into small drops at the height of but 1 or 2 meters above it. (b) Assume the position of the sun proper for the formation of a horizontal bow. This condition also was fulfilled in the cases under discussion.

Now, suppose the eye of the observer placed at the origin, O, of the rectangular coordinates x, y, z, in figure 3. Let z be taken vertically upward and y perpendicular to a ray from the sun. For the sake of simplicity, the sun's rays may be regarded as all parallel at any instant; the altitude of the sun may be designated as h, and the direction cosines of the sun's rays may be λ, μ, ν, where

$\lambda = \cos h$, $\nu = -\sin h$. The equation of the sheet of mist having a height a above O, will be $z = a$.

According to the geometrical theory of the rainbow, the drops reflecting any spectrum color of the rainbow to the eye must lie in the surface of a cone whose apex is at the observer's eye, O, whose axis is parallel with the ray from the source of light, S, and whose semiapical angle is the supplement of the angle of minimum deviation for that color and for those drops. According to the new theory,[2] which considers diffraction phenomena, the magnitude of the semiapical angle of the cone is slightly different from that of the geometrical theory, and varies with the magnitude of the drops. In either case we shall designate the apical angle of the cone by θ.

The equation of the cone becomes

$$\sqrt{(x^2+y^2+z^2)} \cos \theta = (\lambda x + \nu z)$$

Dispersion phenomena must occur in the drops above the lake surface when the sun shines upon them. The phenomena must be observed along the curve of intersection of the semicone and the plane of the sheet of drops. The equation of the curve of intersection is

$$\sqrt{(x^2+y^2+a^2)} \cos \theta = (\lambda x + \nu a),$$
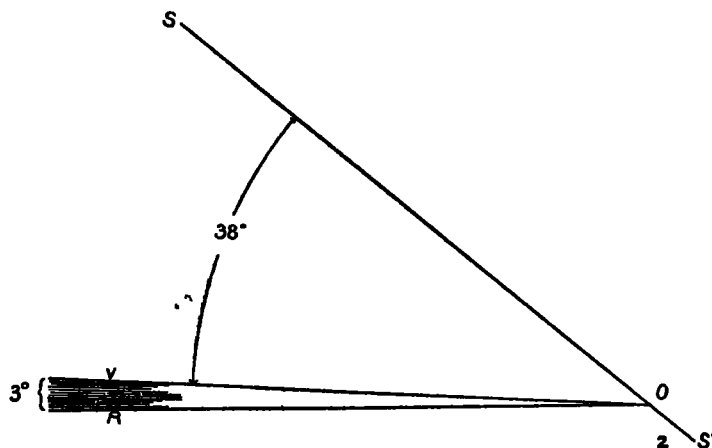
in the plane $z = a$.

FIG. 2.—Optical circumstances favoring a horizontal rainbow on December 8, 1913.

From this equation we know

(I) The curve is (a). An ellipse, when $\cos^2\theta > \lambda^2$, $a < 0$, and the equation becomes

$$\frac{\left(x + \dfrac{a\lambda\nu}{\cos^2\theta - \lambda^2}\right)^2}{\dfrac{a^2 \cos^2\theta \sin^2\theta}{(\cos^2\theta - \lambda^2)^3}} + \frac{y^2}{\dfrac{a^2 \sin^2\theta}{\cos^2\theta - \lambda^2}} = 1,$$

(I) (b) A single point, when $\cos^2\theta > \lambda^2$, $a = 0$, its coördinate being

$$x = 0, y = 0,$$

(I) (c). Vanishes when $\cos^2\theta > \lambda^2$, $a > 0$.

(II) The curve is (a): A parabola, when $\cos^2\theta = \lambda^2$, $a < 0$, and the equation becomes

$$y^2 = -2b \tan \theta \, (x + b \cot 2\theta)$$

[2] Airy, Trans. Camb. phil. soc., 1838, v. 6, p. 379, and 1848, v. 8, p. 595. Also papers by Boitel, Larmor, Mascart, L. Lorenz, Pernter, Aichi, and T. Tanakadate.

(II) (b). A straight line, when $\cos^2\theta = \lambda^2$, $a = 0$, and the equation becomes

$$y = 0.$$

(II) (c). Vanishes, when $\cos^2\theta = \lambda^2$, $a > 0$.

(III) (a), and (III) (c). The curve becomes one branch of a hyperbola, when $\lambda^2 > \cos^2\theta$, $a \gtrless 0$, and the equation is

$$\frac{\left(x - \frac{a\lambda\nu}{\lambda^2 - \cos^2\theta}\right)^2}{\frac{a^2\cos^2\theta\sin^2\theta}{(\lambda^2-\cos^2\theta)^2}} - \frac{y^2}{\frac{a^2\sin^2\theta}{\lambda^2-\cos^2\theta}} = 1$$

(III) (b). Two straight lines passing through the origin, when $\lambda^2 > \cos^2\theta$, $a = 0$, and the equations are

$$x = \pm y \frac{\cos\theta}{\sqrt{\lambda^2 - \cos^2\theta}}.$$

Thus, we see that in order to observe the horizontal rainbow it is convenient to have the eye not lower than the sheet of water drops.
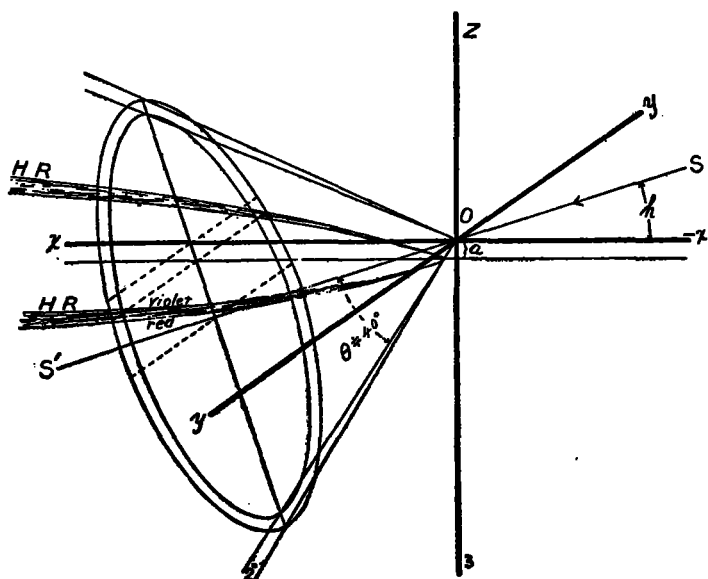

FIG. 3.—Diagrammatical explanation of the horizontal rainbow.

Suppose the eye is on a level with the sheet of drops. At sunrise, since $\lambda^2 = 1 > \cos\theta$ and $\lambda^2 - \cos^2\theta = \sin^2\theta$, the rainbow must appear in two straight lines, each making the angle θ with the ray which enters the eye of the observer. As the altitude of the sun increases λ decreases, and consequently the angle between the two straight lines becomes smaller and smaller. At the instant when $\lambda = \cos\theta$ the two coincide and at the next moment the rainbow vanishes. As the sun travels from east to west the axis of the cone shifts from left to right of the observer.

Next consider the case when the eye of the observer is above the sheet of water drops. First the horizontal rainbow is of the form of one branch of an hyperbola. Gradually the vertex approaches the observer and the aperture of the curve diminishes, its axis shifting from left to right. At the instant when $\cos^2\theta = \lambda^2$ the curve becomes a parabola and at the next moment it becomes an ellipse. Thenceforth its major and minor axes diminish, but they do not vanish unless the mist disappears. After the sun passes the meridian the dimension of the bow increases again, repeating the same changes but in the inverse order.

In order that the horizontal rainbow may be distinctly visible, the angular distribution of the drops must be somewhat dense, so that as the observer rises higher the figure becomes less distinct. For the above reason, and since the eyes of the observer are situated in general a few meters above the surface of the lake, the hyperbolic or parabolic bow must be one or two sensibly straight lines when the horizontal rainbow appears at a great distance from the observer. Since light undergoes dispersion, there must be tape-like colored bands along these two straight lines. This is why the colors in the horizontal rainbow appear radially and in the form of a band. The horizontal rainbow appears on clear days because only on clear days can there be drops with the sun shining upon them. Thus, all the principal facts are explained.

Next we will calculate the breadth of the rainbow and will find the position of the sun favorable to the actual production of a horizontal rainbow. Let l, m, n be the direction cosines of the beam from any drop near the surface of the lake to the eye. Then

$$\cos\theta = l\lambda + n\nu$$

or

$$l = \frac{1}{\lambda}(\cos\theta - n\nu)$$

We will distinguish all symbols belonging to the violet and the red rays of the bow by inferiors v and r, respectively. The angle ϕ between the violet and the red rays is expressed by the formula

$$\cos\phi = l_v l_r + m_v m_r + n_v n_r.$$

Since

$$l_v = \frac{1}{\lambda}(\cos\theta_v - n_v\nu), \qquad l_r = \frac{1}{\lambda}(\cos\theta_r - n_r\nu),$$

$$m_v^2 = 1 - l_v^2 - n_v^2 = 1 - \frac{1}{\lambda^2}(\cos\theta_v - n_v\nu)^2 - n_v^2,$$

$$m_r^2 = 1 - l_r^2 - n_r^2 = 1 - \frac{1}{\lambda^2}(\cos\theta_r - n_r\nu)^2 - n_r^2,$$

$$\cos\phi = \frac{1}{\lambda^2}[(\cos\theta_v - n_v\nu)(\cos\theta_r - n_r\nu)$$

$$- \sqrt{\{\lambda^2 - (\cos\theta_v - n_v\lambda)^2 - n_v^2\lambda^2\}\{\lambda^2 - (\cos\theta_r - n_r\lambda)^2 - n_r^2\lambda^2\}}]$$

$$+ n_v n_r.$$

λ becomes zero when the sun is at the zenith, but this does not occur in our latitudes. If λ becomes zero, then $\cos\phi = 1$ or $\phi = 0°$.

In the actual case, since $\theta_r \doteqdot 42°$ and $\theta_v \doteqdot 40°$, and since the rays from the drops are sensibly horizontal, $n\nu$ may be neglected. Thus we may put

$$\cos\phi = \frac{1}{\lambda^2}[\cos\theta_r \cos\theta_v + \sqrt{(\lambda^2 - \cos^2\theta_v)(\lambda^2 - \cos^2\theta_r)}]. \qquad (1)$$

If $\lambda = 1$, or the sun is on the horizon,
$$\cos\phi = \cos\theta_r \cos\theta_v + \sin_r\nu \sin\theta_v,$$
$$= \cos(\theta_r - \theta_v),$$
$$\doteqdot \cos 2°$$
and
$$\phi \doteqdot 2°.$$
If $\lambda = \cos\theta_v$
then $\cos\phi = \dfrac{\cos\theta_r}{\cos\theta_v}$
$$= \cos 16° 44'$$
and
$$\phi = 16° 44'.$$

This shows that the breadth of the rainbow must be some value between 2° and 16° 44'. The values θ_r and θ_v here used are rough approximations, hence the quantities obtained only show the order of magnitude. The exact values of θ_r and θ_v vary with the magnitude of the water drops. If the drops are small the angles diminish even to the value $\theta_v = 38°$. In such a case the breadth of the bow becomes smaller than the value shown above.

Next we shall calculate the breadth of the rainbow in the actual case. The position of the observer on November 3, 1912, was

longitude 138° 7' E. of Greenwich.
latitude 36° 3' N.

Declination of the sun at 9, 10, and 11 a. m. was −14° 53', −14° 54', and −14° 55', respectively, and the corresponding computed altitudes of the sun were

$$h_9 = 27° 45', \quad h_{10} = 34° 39', \quad h_{11} = 38° 29'.$$

Introducing these values in equation (1) we get

$$\phi_9 = 4° 4', \quad \phi_{10} = 5° 57', \quad \phi_{11} = 10° 44'.$$

The maximum altitude (at the meridian transit) of the sun on that day occurred at $11^h 31^m 27^s$ civil time, and its amount was 39° 1' 36''.

If we put $h = 39°$ or $\lambda = \cos 39°$, then we get from equation (1)

$$\phi = 14° 0'.$$

Since the observation was a rough one, this value for ϕ must be looked upon as fairly coinciding with the actual value, that is, $\phi = 15°$.

On December 8, 1913, the horizontal rainbow was observed at 8 a. m. At that time the sun's altitude was 11° 23' for which $\phi = 1° 28'$, or nearly one-half its observed value of 3°. At 9 a. m. of that day the sun's altitude became 19° 17', and hence $\phi = 3° 17'$.

Thus we see that if the time of observation was somewhat later than 8 a. m. (in the observer's report the time is said to be about 8 a. m., so the number of minutes is naturally obscure), then the calculated value of ϕ becomes greater than 1° 28', and if the true width would be somewhat less than observed, then the theory may coincide with the facts.

The angle SOV in figure 2 can be calculated. Since

$$\cos 40° = l_v \lambda + n_v \nu$$
$$\doteqdot l_v \lambda,$$

hence

$$l_v = \frac{\cos 40°}{\lambda} = \frac{\cos 40°}{\cos 11° 23'} = \cos 38° 11'$$

at 8 a. m. Thus value agrees very well with the observed value 38°.

Thus there is sufficiently close agreement between theory and facts to determine the true cause of the horizontal bow. One point remains to be noticed, however. The observer tells us that on November 3 the rainbow vanished at 11 a. m. How can it vanish? There must be two explanations. The first of these is that the sheet of drops might vanish with the increase of air temperature; the second is that the drops did not dissolve but that the sun became so high that its altitude exceeded the apical angles of the cones for both the violet and the red rays. At first thought it seems likely that the altitude of the sun could not increase to a sufficient degree to cause the rainbow to vanish, because θ_v is nearly equal

to 40° and the calculated maximum altitude of the sun is 39° 2'. But as before remarked, the above value of θ is a rough approximation. Sometimes when the drops are very small, e. g. of a radius less than 0.025 mm., the the value of θ may become less than 38°. Since the mist was very thin, such small drops may have existed. If so, then, when the sun reached the proper altitude, the bow would become invisible to an observer whose eyes were on a level with the sheet of drops. To one whose eyes were above the sheet of drops, however, the bow might still be visible, though it would have become somewhat indistinct. This was proved by the above given observation.

POSTSCRIPT, JULY 12, 1914.

On January 30, 1914, Mr. Katsuji Nakamura, of the Central Meteorological Observatory, Tokyo, happened to observe a horizontal rainbow in the moat of the observatory. After that, this often appeared in the morning of calm winter days. The account of this is given in the Journal of the Meteorological Society of Japan, 33d year, No. 6. He and I made observations of the angular breadth of the bow, the difference of the departures of the violet and red ends of the bow from the direction of the sun's ray passing through the observer's eye, the altitudes of the sun, etc., and found a good agreement with the theory given in the text. Since I have written about the horizontal rainbow in the Journal of the Meteorological Society of Japan, many reports came from observers stating the occurrences of horizontal bows on Lake Suwa. In some cases two branches of a bow were seen at the same time. I have learned from Prof. Nagaoka, of the Imperial University, Tokyo, that he observed a horizontal rainbow on the ice sheet of the Sinobazu Pond, in Tokyo. On that day the weather was very clear, but a thin sheet of mist was observed on the ice surface. Prof. S. Nakamura, of the same university, observed a similar phenomenon on the moat in front of the imperial palace in Tokyo. On this occasion the weather was also clear, and there was no mist perceived over the moat, but he observed some oily specks on the surface of the water. All these phenomena occurred in the morning of very calm and bright winter days.

I have heard from Prof. Nagaoka that J. C. Maxwell observed a colored bow on the frozen surface of the ditch which surrounds St. John's College, Cambridge.[3] It occurred on the 26th of January, 1870, at about noon. He measured with a sextant and found that the angular distance of the bright red of the bow from the sun's ray was 41° 50', and that of the bright blue 40° 30'. He considered the bow to be produced by water drops on the ice surface. Bows formed by ice crystals are seen on the same side as the sun and not on the opposite side. These angles are somewhat smaller than the values for an ordinary rainbow. This fact was also found by us. The following table shows the values of the angular apertures of the bows observed in Tokyo:

Date.	Jan. 30, 1914, 9:50 a. m.	Jan. 30, 1914, 10:25 a. m.	Feb. 17, 1914, 9:44 a. m.
	° '	° '	° '
Red..................................	41 35	42 36	41 52
Violet...............................	38 51	39 53	40 02
Difference...........................	2 41	2 43	1 50

Maxwell left two questions unsolved, one of which was why are the angular apertures of the horizontal bow

[3] *Maxwell.* On a bow seen on the surface of ice. Edinb. Roy. soc. proc., v. 7; also see Scientific Papers, II, p. 160.

smaller than those of the ordinary bow; the other question was that how a drop of water can lie upon ice without wetting it and losing its shape altogether. In the light of the modern science these two questions seem to be clearly answered, as shown in the text of the present paper, taking the following facts into consideration: first, the water drops are very small ones, and, second, they float in the thin stratum of air in contact with the ice surface but [they do] not lie on the surface. According to the diffraction theory of the rainbow, the angular aperture of the bow becomes small when the droplets producing the bow are very small. In many cases they are invisibly small. That the droplets are floating in air, but not lying on the surface of ice or water, can easily be seen from the fact that the bow appears on the water surface as well as on the ice surface, and also that it occurs always on calm and bright mornings in the cold season. The following fact also supports the idea: On the morning of March 17 we observed a horizontal bow in the moat of our observatory. At about 9:20 a. m. we saw wind that came over the water surface from the west. The bow became faint when the head of ripples arrived at the bow, and gradually it vanished away as the wind became stronger.

From early times people in Suwa have been well acquainted with the phenomenon. They call the phenomenon "Ohikari" which means literally a holy shine, and take it to be a foretoken of the coming change of the weather. Indeed, many times when we observed a horizontal rainbow in the moat of this observatory we experienced rain or storm one or two days after. The reason of this, in my opinion, must be as follows: On the Pacific side of Japan the weather in winter is generally clear and the northwesterly monsoon prevails every day. On the appearing of a cyclone in the west the monsoon is disturbed by the easterly or southeasterly winds flowing into the cyclonic center. Hence there then prevails a calm. Considering the atmospheric pressure, this calm corresponds to the high pressure over Japan, which is followed by a cyclone. Such a calm is always favorable for the formation of a horizontal rainbow. Thus the calm, as well as the horizontal rainbow, are in many cases the foretokens of the coming cyclone.

OBSERVATIONS OF HORIZONTAL RAINBOWS.[1]

By Katsuji Nakamura.

[Dated Central Meteorological Observatory, Tokyo.]

The author of the present note had favorable opportunities of observing the horizontal rainbow in the moat near the entrance of Central Meteorological Observatory, which is situated in the compound of the old castle of Tokyo. The following lines contain short descriptions of the phenomena and of the weather conditions that then prevailed:

1. *Horizontal rainbow on January 30, 1914.*—On the morning of January 30, 1914, a greater part of the surface of the moat was covered with a thin coating of ice. The sky was cloudless, and the air was so calm that we scarcely felt even the quivering of the leaves of trees and grasses. We observed the rainbow from 9:30 a. m. until 10:40 a. m. before the ice began to melt.

When we stood at A (a point on the bridge) in figure 1, turning our back to the sun, we saw the rainbow on the

left and downward as at DH, but its colors were not distinct. Then as we moved from A toward B, C, . . . the far side of the rainbow appeared to shift from D toward R and E, . . . and its colors gradually became vivid.

As seen from F, a point higher than A, B, M, C, etc., we also saw the rainbow toward KF, but its colors were not so distinct as when seen from the other places, A, B, etc. At G, one of the highest places, we saw that the bow was lying toward LG, its color becoming fainter. But when standing at O, a point having an equal height with G, we could not see the bow.

From what we have stated above we see that G is one of the limits of visibility of the rainbow on this morning.
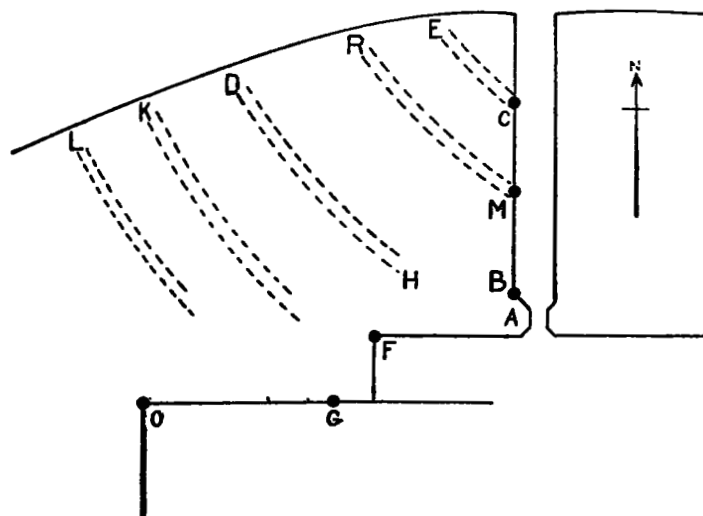

FIG. 1.—Plan of moat of Central Meteorological Observatory, Tokyo.

On the ice surface the rainbow was more clearly visible than on the water.

We give here the widths of the rainbow that Dr. S. Fujiwhara observed with a sextant:

9:50 a. m.:	°	′	10:25 a. m.:	°	′
Red	41	35	Red	42	36
Violet	38	54	Violet	39	53
Width	2	41	Width	2	43

Of course these values are rough approximations.

2. *The rainbow observed on February 17, 1914.*—On this morning upper clouds thinly covered the sun and the sky and it was so calm that it grew rather hazy all around and there was no ice in the moat.

We observed the rainbow, on this morning also, at the same places, A, B, M, C, etc., in figure 1. Its color as seen from B was more vivid than when seen from A, and clearer as seen from M than from B. When seen from C it was not so clear as when seen from M, that is to say, from M the bow was clearer than from other points and the color became thin by standing away from the place M. Besides this, even on the same zone MR, the color of the bow at R was very clear and became gradually faint toward M.

We give here the results of Dr. Fujiwhara's measurements:

9:44 a. m.:	°	′
Red	41	52
Violet	40	02
Width	1	50